DYNAMIC DEVELOPMENT OF A RADIAL CRACK ZONE

DURING A CAMOUFLET EXPLOSION

E. N. Sher

UDC 622.235

In describing the destructive effect of a camouflet explosion in solid media there is extensive use of zonal models [1-4] separated normally into a grinding zone near the charge, an intermediate zone of radial cracks, and an outer zone of elasticity. The difference in [1-4] in describing zones of radial cracks consists in different failure criteria for elastic material by an outer zone of radial cracks. Here a force approach is used for critical tangential stresses [1, 2], taking account from an energy viewpoint of energy consumed in crack formation [3, 5] and a kinematic approach [4].

In [6-8] it is suggested that the zone of radial cracks and outer elastic zone are considered in a single elastic arrangement. Crack growth is determined by the dependence of crack velocity on stress intensity factor at its tip, which is a rating characteristic for broken material. The scheme suggested is easily realized in a quasistatic arrangement for the problem of exploding a cord charge, since a broad class of problems about equilibrium for a radial crack system in an elastic plane has been studied. Difficulties in obtaining accurate dynamic solutions for this problem compel us to find approximate solutions. Such a solution on the approach of a large number of cracks is obtained if again we turn to separating the radial and outer elastic zones. A similar approach for the static problem is used in [9].

In an elastic zone for displacements u(r, t) for > $\ell(t)$

$$\frac{1}{a^2}\frac{\partial^2 u}{\partial t^2}=\frac{\partial^2 u}{\partial r^2}+\frac{1}{r}\frac{\partial u}{\partial r}-\frac{u}{r^2},\ a^2=\frac{E(1-\nu)}{(1+\nu)(1-2\nu)\rho_0}, \tag{1}$$

where a is longitudinal wave velocity; E is Young's modulus; ν is Poisson's ratio; ρ_0 is material density.

In the zone of radial cracks for columnar elasticity with $r_0(t) < r < \ell(t)$ an equation is fulfilled [2]

$$\frac{1}{a_1^2}\frac{\partial^2 u_1}{\partial t^2}=\frac{\partial^2 u_1}{\partial r^2}+\frac{1}{r}\frac{\partial u_1}{\partial r},\ a_1^2=\frac{E}{(1-\nu^2)\rho_0}. \tag{2}$$

For stress tensor components in this zone

$$\sigma_{r1}=\frac{E}{(1-\nu^2)}\frac{\partial u_1}{\partial r},\ \sigma_{\theta 1}=0. \tag{3}$$

We consider the problem with the following boundary conditions: at the inner boundary of the radial crack zone with $r = r_0(t)$

$$\sigma_r=-p(t); \tag{4}$$

and at the crack front with $r = \ell(t)$

$$u_1=u; \tag{5}$$

$$\sigma_{r1}+\rho_0\dot{l}\,\frac{\partial u_1}{\partial t}=\sigma_r+\rho_0\dot{l}\,\frac{\partial u}{\partial t}; \tag{6}$$

$$2\gamma_0 n=V-V_1+\frac{1}{2}(\sigma_r+\sigma_{r1})\left(\frac{\partial u_1}{\partial r}-\frac{\partial u}{\partial r}\right)\quad (\dot{l}\neq 0), \tag{7}$$

$$2\gamma_0 n=V-V_1+\sigma_r\left(\frac{\partial u_1}{\partial r}-\frac{\partial u}{\partial r}\right)\quad (\dot{l}=0).$$

Equations (5) and (6) are expressions of continuity for mass flows and a pulse in Lagrangian variables, and Eq. (7) expresses the rule for transfer of energy across the

Novosibirsk. Translated from Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, No. 1, pp. 164-167, January-February, 1988. Original article submitted January 20, 1986.

crack front. In these equations V and V_1 are bulk density of elastic energy; $\gamma_0(\dot{\ell})$ is density of surface energy going into crack formation; n is the number of cracks per unit length of their front. With uniformly distributed N cracks $n = N/(2\pi\ell)$. An energy conservation equation is given in a similar form in [4]. Here only term $2\gamma_0 n$, determining energy dissipation going into failure, is concretized, and the energy condition for an immobile front with $\dot{\ell} = 0$ is specified.

Function γ_0 is connected with stress intensity factor by the relationship [10]

$$2Eb^2\gamma_0 R(\dot{l}) = (1+\nu)\dot{l}^2 K_I^2 \sqrt{1-\dot{l}^2/a^2},$$
$$R(c) = 4\sqrt{1-c^2/a^2}\sqrt{1-c^2/b^2} - (2-c^2/b^2)^2, \tag{8}$$

where b is transverse wave velocity; K_I is stress intensity factor. Taking account of this connection it is possible to assume γ_0 is a known function for crack front velocity $\dot{\ell}$ if rating relationship $K_I(\dot{\ell})$ is known. In this case the rule for movement $\ell(t)$ is found with solution of the stated problem. If $\ell(t)$ is prescribed, then (7) makes it possible to determine $\gamma_0(t)$ and $K_I(t)$ according to (8).

As an example of the latter arrangement we carry out solution of a self-modelling problem of expansion of a columnar elasticity zone with constant velocity from zero size under the effect of external pressure $p = p_0 t/t_0$. This problem is a zonal approximation of the self-modelling problem for growth at constant velocity of a star of a considerable number of cracks under the action of internal concentrated forces. It is not resolved in the general case, and only the case of two cracks is considered.

Condition (4) for this problem is written in the form

$$\sigma_r = -p_0 t_0/t \text{ with } r = v_0 t. \tag{9}$$

With this loading, displacement should be a uniform function of a zero power relative to variables r and t. A general solution of this form of Eq. (2) is the function

$$u_1(r/a_1 t) = a^0 + a^1 \ln (z - \sqrt{z^2-1}),\ z = a_1 t/r. \tag{10}$$

Taking account of (3), from (9) we obtain $a^1 = -p_0 t_0 v_0 (1-\nu_2)\sqrt{1 - v_0^2/a_1^2}/E$. For velocity and radial strength at the crack front with $r = ct - 0$

$$v_1 = \frac{\partial u_1}{\partial t} = \frac{p_0(1-\nu^2)t_0 v_0\sqrt{1-v_0^2/a_1^2}}{Et\sqrt{1-c^2/a_1^2}},$$
$$\sigma_{r1} = -\frac{p_0 t_0 v_0\sqrt{1-v_0^2/a_1^2}}{ct\sqrt{1-c^2/a_1^2}}. \tag{11}$$

Solution of Eq. (1) in the elastic zone is found in the form $u = \partial\varphi/\partial r$. For potential φ we write an equation similar to (2): $\frac{\partial^2\varphi}{\partial r^2} + \frac{1}{r}\frac{\partial\varphi}{\partial r} = \frac{1}{a^2}\frac{\partial^2\varphi}{\partial t^2}$. Solution of this (with the required degree of uniformity and providing conversion of displacement u to zero with $r = at$, which is determined by the initial repose condition [11]) has the form $\varphi(r, t) = A_1 r(z \ln (z - \sqrt{z^2-1}) + \sqrt{z^2-1})/a$, $z = at/r$. In this way

$$u(r, t) = A_1\sqrt{z^2-1}/a. \tag{12}$$

For velocity and radial stress ahead of the crack front with $r = ct + 0$

$$v = \frac{\partial u}{\partial t} = \frac{A_1}{ct}\frac{1}{\sqrt{1-c^2/a^2}},$$
$$\sigma_r = -\frac{A_1\rho_0 a^2(1-2\nu+\nu c^2/a^2)}{tc^2(1-\nu)\sqrt{1-c^2/a^2}}. \tag{13}$$

Unknown constant A_1 in (12) and (13) should be determined from conditions at the crack front. Condition (5) is satisfied by selecting constants a^0 in (10), and from (6) and (11), (13) we find

$$A_1 = \frac{p_0 t_0 v_0\sqrt{1-v_0^2/a_1^2}\sqrt{1-c^2/a_1^2}\,c(1-\nu)}{\rho_0 a^2(1-2\nu)\sqrt{1-c^2/a^2}}.$$

Equality (7) for a determinate form of movement makes it possible to find γ_0, and using (8) to find $K_I(t)$:

$$K_I = \frac{Q\sqrt{N}}{2\sqrt{\pi ct}} \frac{b\sqrt{(1-\nu)R(c)}\sqrt{1-v_0^2/a_1^2(1-c^2/(2b^2))}}{(1-c^2/a^2)^{3/4}}, \qquad (14)$$

$$Q = 2\pi v_0 t_0 p_0/N.$$

With the limiting conversion of v_0 and c to zero expression, (14) is transformed into an asymptotic of the solution for the static problem of stresses for a star of N cracks with length ℓ by internal forces Q with large N [12]: $K_I = Q\sqrt{N}/(2\sqrt{\pi\ell})$.

The method suggested makes it possible also to solve static problems of equilibrium for an elastic half-space with a circular or a spherical cavity with a prescribed internal pressure and presence of a radial system of a large number of cracks. At the front of the crack zone a continuity condition is fulfilled for σ_r and u, and in addition energy condition (7) is fulfilled. In using this method to solve the Bowie problem results are obtained agreeing asymptotically with those known with a large number of cracks [12, 13].

The most important advantage of this approach is the possibility of realizing numerical solution of problems for breaking a brittle material during blasting from the position of fracture mechanics in a general scheme of a zonal approach to the problem in question. With this approach it is possible to take account of failure for zones of different structure close to the explosive charge and to construct adequate mathematical models of the problem.

LITERATURE CITED

1. V. N. Rodionov, A. N. Romashev, et al., Mechanical Effect of an Underground Explosion [in Russian], Nedra, Moscow (1971).
2. S. S. Grigoryan, "Some questions of mathematical theory for deformation and failure of hard rocks," Prikl. Mat. Mekh., 31, No. 4 (1967).
3. G. P. Cherepanov, Mechanics of Brittle Fracture [in Russian], Nauka, Moscow (1974).
4. V. P. Nikolaevskii, "Limiting velocity of the failure front in dynamic overloading of brittle materials," Moscow (1979) (Preprint, IFZ, Akad. Nauk SSSR, No. 123).
5. L. I. Slepyan, "Brittle fracture wave," Inzh. Zh. MTT, No. 4 (1968).
6. E. N. Sher, "Estimate of the crushing effect on an elongated charge in a brittle material," Fiz.-Tekh. Probl. Razrab. Polez. Iskop., No. 1 (1975).
7. E. N. Sher, "Example of calculating the movement of radial cracks formed during blasting in a brittle material in a quasistatic approximation," Fiz.-Tekh. Probl. Razrab. Polez. Iskop., No. 2 (1982).
8. P. A. Martynyuk and E. N. Sher, "Estimate of the sizes of radial crack zones forming during a camouflet explosion of a cord charge in a brittle material," Zh. Prikl. Mekh. Tekh. Fiz., No. 4 (1984).
9. Ya. V. Voitishek and L. I. Slepyan, "Hydrodynamic model of punching a brittle plate," Fiz.-Tekh. Probl. Razrab. Polez. Iskop., No. 3 (1985).
10. E. N. Sher, "Energy condition at the nose of a nonsteady crack," Zh. Prikl. Mekh. Tekh. Fiz., No. 3 (1969).
11. S. L. Sobolev, "Some questions of vibration propagation theory," in: Differential and Integral Equations of Mathematical Physics, F. Frank and R. Mises (eds.), [Russian translation], ONTI, Leningrad–Moscow (1937).
12. E. N. Sher, "A case of equilibrium for a radial crack system," Zh. Prikl. Mekh. Tekh. Fiz., No. 5 (1974).
13. F. Ouchterlony, Fracture Mechanics Applied to Rock Blasting, Stockholm (1973) (Rept/Swedish Detonic Res. Found.; DS 1973: 29).